Theory of machinery

Chapter three

Velocity analysis

By

Laith Batarseh

Velocity analysis

Theory of machinery

VECTOR ALGEBRA

•If we assume that the dimension ϴ is represented by a unit vector U_Θ, then the derivative of the unit vector ($\dot{U}_\theta$) can be found as:-

$$\dot{U}_\theta = \frac{dU_\theta}{dt} = \lim_{\Delta t \to 0} \frac{U_{\theta+\Delta\theta} - U_\theta}{\Delta t}$$

Assume a position vector **r** : $\mathbf{r} = rU_\theta$

Take derivative with respect to time: $\dot{\mathbf{r}} = (r)(\omega)\dot{U}_\theta + \dot{r}\,U_\theta$

But $\dot{r} = 0$ and so:

$$\dot{\mathbf{r}} = (r)(\omega)\dot{U}_\theta$$

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VECTOR ALGEBRA

•According to the previous derivation, if the vector $\mathbf{dU_\theta}$ represent a link then its derivative is found as :

$$\frac{d(dU_\theta)}{dt} = (d)(\omega)\dot{U}_\theta$$

Note that:- assume $U_{\Theta 1} = \cos(\Theta_1)i+\sin(\Theta_1)j$ and $U_{\Theta 2} = \cos(\Theta_2)i+\sin(\Theta_2)j$:-

$$\dot{U}_{\theta 1} \bullet U_{\theta 1} = 0$$

$$U_{\theta 1} \bullet \dot{U}_{\theta 2} = \sin(\theta_1 - \theta_2)$$

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4-BAR MECHANISM

LOOP CLOSURE EQUATION

$$d_2 U_{\theta 2} + d_3 U_{\theta 3} = d_1 U_{\theta 1} + d_4 U_{\theta 4}$$

Derivative

$$d_2\omega_2 \dot{U}_{\theta 2} + d_3\omega_3 \dot{U}_{\theta 3} = d_4\omega_4 \dot{U}_{\theta 4}$$

Dot product both sides by $\mathbf{U_{\theta 3}}$ **to eliminate** $\boldsymbol{\omega_3}$

$$d_2\omega_2 \sin(\theta_3 - \theta_2) + 0 = d_4\omega_4 \sin(\theta_3 - \theta_4)$$

Solve for ω_4:- $\omega_4 = \dfrac{d_2\omega_2 \sin(\theta_3 - \theta_2)}{d_4 \sin(\theta_3 - \theta_4)}$

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4-BAR MECHANISM

FIND ω_3

to find ω_3, dot product both sides of derivative equation by $\mathbf{U_{\theta 4}}$ **to eliminate ω_4** :

$$d_2\,\omega_2\,\sin(\theta_4-\theta_2) + d_3\,\omega_3\,\sin(\theta_4-\theta_3) = 0$$

solve this equation for ω_3:-

$$\omega_3 = -\frac{d_2\omega_2\sin(\theta_4-\theta_2)}{d_3\sin(\theta_4-\theta_3)}$$

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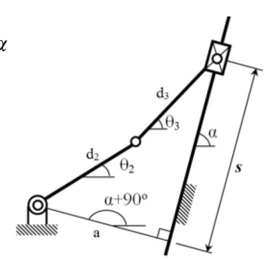
SLIDER CRANK MECHANISM

LOOP CLOSURE EQUATION

$$d_2U_{\theta 2} + d_3U_{\theta 3} + aU_{\alpha+90} = sU_{\alpha}$$

Derivative

$$d_2\omega_2\,\dot{U}_{\theta 2} + d_3\omega_3\,\dot{U}_{\theta 3} = \dot{S}\,U_{\alpha}$$



Dot product both sides by $\mathbf{U_{\theta 3}}$ **to eliminate ω_3**

$$d_2\omega_2\sin(\theta_3-\theta_2) + 0 = \dot{S}\cos(\theta_3-\alpha)$$

Solve for $\dot{S}$:- $\quad \dot{S} = \dfrac{d_2\omega_2\sin(\theta_3-\theta_2)}{\cos(\theta_3-\alpha)}$

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SLIDER CRANK MECHANISM

FIND ω_3

to find ω_3, dot product both sides of derivative equation by **$U^{\cdot}_{\alpha}$ to eliminate $S^{\cdot}$**:

$$d_2\,\omega_2\cos(\theta_2-\alpha) + d_3\,\omega_3\cos(\theta_3-\alpha) = 0$$

solve this equation for ω_3:- $\omega_3 = -\dfrac{d_2\omega_2\cos(\theta_2-\alpha)}{d_3\cos(\theta_3-\alpha)}$

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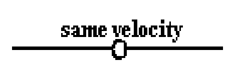
Velocity analysis

[redacted]oth bodies has the same velocity . This point is called instant center of velocity

- a normal axis through this point represent an axis of rotation common to the two bodies



- In a mechanism with n links C(№ of instant centers) is found as

$$C = \frac{n(n-1)}{2}$$

Kennedy's rule : any three bodies have three instant centers of velocity that lie on the same straight line

Velocity analysis

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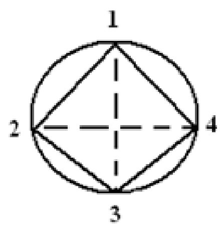
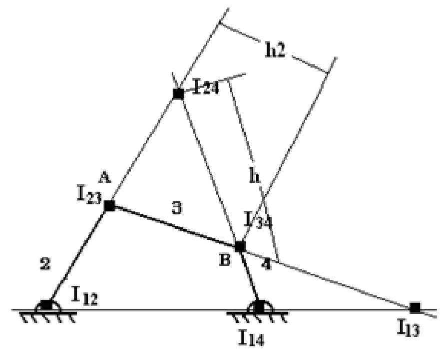
I .C of four bar

$$C = \frac{n(n-1)}{2} = \frac{4(3-1)}{2} = 6$$





Velocity analysis

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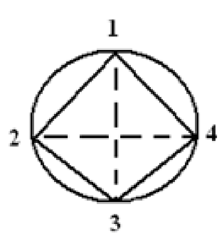
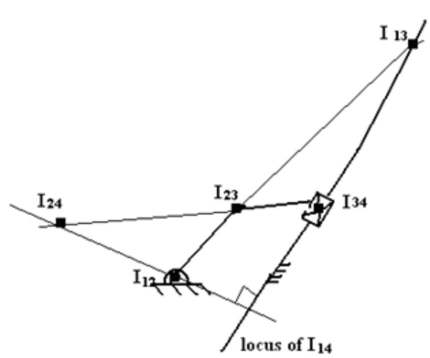
I .C of slider crank

$$C = \frac{n(n-1)}{2} = \frac{4(3-1)}{2} = 6$$





Velocity analysis

Theory of machinery

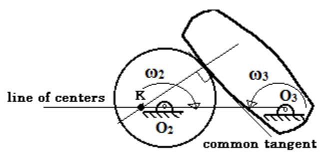
SPEED RATIO

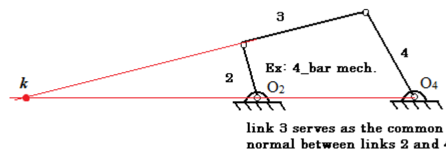
speed ratio is the ratio between motions of rotating links with other rotating or translational links

There are two common cases of finding the speed ratio:

Between two rotating links have a direct contact

Between to links have a common link acts as normal to the two links





Velocity analysis

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SPEED RATIO

- Draw line of centers from the rotating axes ($O_2 - O_4$)
- Draw extension line for link number 3 until it intersect the line of center at point **k**.
- The speed ratio: will be found using the following equation

$$\frac{\omega_4}{\omega_2} = \frac{O_2 k}{O_4 k}$$

Where:-

- $\boldsymbol{O_2 k}$ is the straight distance measured between the points $\boldsymbol{O_2}$ and $\boldsymbol{k}$.
- $\boldsymbol{O_4 k}$ is the straight distance measured between the points $\boldsymbol{O_4}$ and $\boldsymbol{k}$.

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SPEED RATIO

First case

- Draw line of centers from the rotating axes ($O_2 - O_3$)
- Draw a tangent from the contact point
- Draw a line start from the tangency point perpendicular to the tangent line and intersect the line of centers at a certain point. Call this point **k**.
- The speed ratio: will be found using the following equation

$$\frac{\omega_3}{\omega_2} = \frac{O_2 k}{O_3 k}$$

Where:

- ***O_2k*** is the straight distance measured between the points ***O_2*** and ***k***.
- ***O_3k*** is the straight distance measured between the points ***O_3*** and ***k***.

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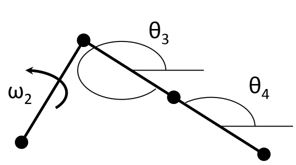
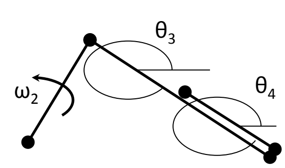
4-BAR MECHANISM

SPECIAL CASES



Stable locking position
$\theta_4 = \theta_3 - \pi$



Unstable locking position
$\theta_4 = \theta_3$

➢**In locking positions speed is too much before θ_3-θ_4 but it becomes zero after that and motion stops**

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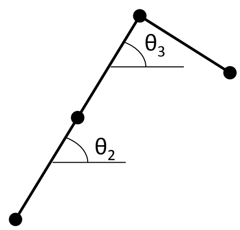
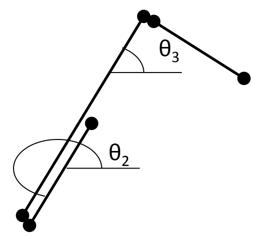
4-BAR MECHANISM

SPECIAL CASES



Limit position
$\theta_2 = \theta_3$



Limit position
$\theta_2 = \theta_3 + \pi$

- **In limit positions speed becomes zero when θ3=θ2**